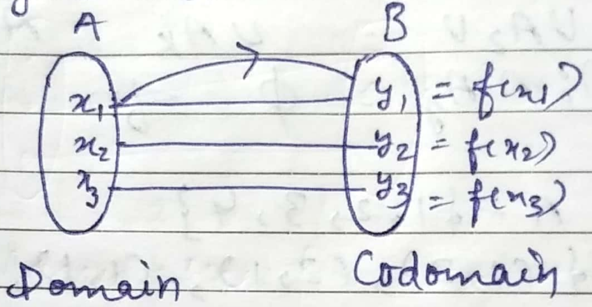
Function :-

Let A and B be two non-empty sets. A function $f: A \to B$ is a rule that assigns to each element x in A a unique element y in B.



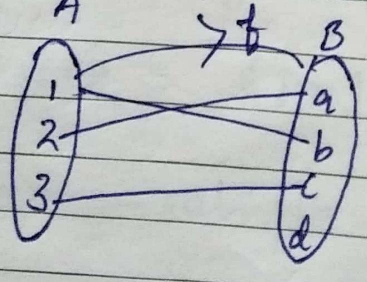
→ if $y = f(x)$ then y is called image of x and x is called Preimage of y

Range of a f^n :- The Set consisting of all the images of the elements of A under the function f is called the range of f. It is denoted by $f(A)$

$$f(A) = \{ f(x) : \text{for all } x \in A \}$$

and $f(A) \subseteq B$.

Ex:- Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$



So Range $f = \{a, b, c\}$

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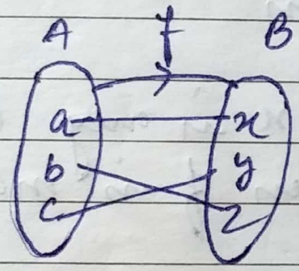
Types of Functions :→

1.) Injective / one-to-one function : →

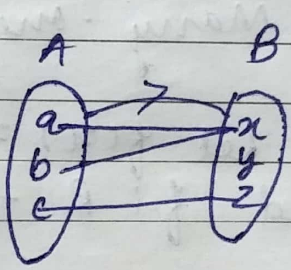
A function $f : A \to B$ is called one-one if no two elements in A corresponds to the same element in B.

i.e $\quad x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

or $\quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$



One-One



Not one-one

Ex:- ① $f(x) = 3x - 1$ is a one-one funct^n $f : R \to R$

Since

$$f(x_1) = f(x_2)$$
$$\Rightarrow 3x_1 - 1 = 3x_2 - 1$$
$$\Rightarrow 3x_1 = 3x_2$$
$$\Rightarrow x_1 = x_2$$

② $f(x) = x^2$ is not one-one where $f : R \to R$

Since

$-1 \neq 1$ but $f(-1) = f(1)$

So $f(x)$ is not one-one f^n.

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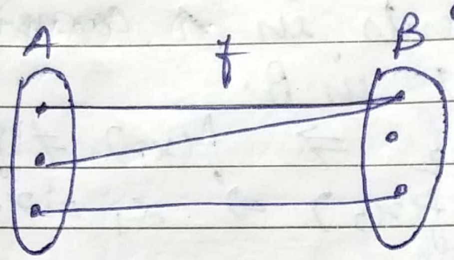
2.) <u>Many-one function</u> :-

A function $f: A \to B$ is said to be many one if and only if two or more elements of A have same image in B.



Many-one function

<u>Ex</u>:- Let $f(x) = x^2$, x is any Real no. and $f: R \to R$, then f is many-one fn

for $x = 1$, $f(1) = 1^2 = 1$
for $x = -1$, $f(-1) = (-1)^2 = 1$

Thus $f(1) = f(-1)$ which shows that two distinct numbers 1 and -1 are assigned to the same no. 1 under f.
$\therefore$ f is many-one.

3.) <u>Into function</u> :-
A function f from A to B is called into function if and only if there exists at least one element in B which is not the image of any element in A, i.e the range of f is a proper subset of codomain of f.

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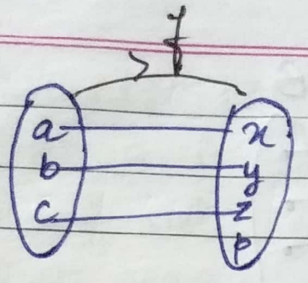


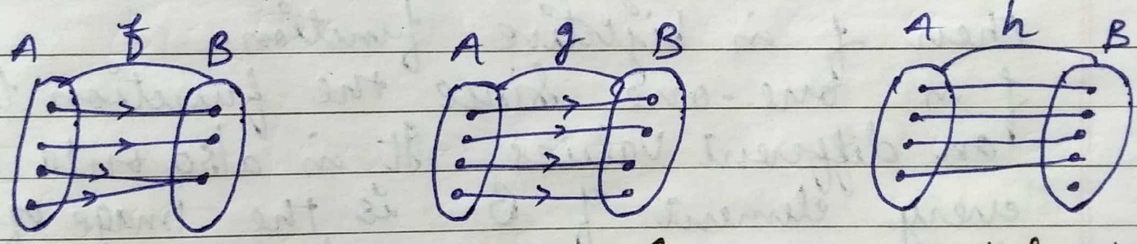
Into function.

4.) Onto Function :-

A function $f : A \to B$ is called onto or Surjective if every element of B is the image of some element of A

i.e if $B = f(A)$ (Range of f)

Notes :- In order to check, whether $y = f(x)$ from set A to B is onto or not, write x in terms of y and see if for every $y \in B$, $x \in A$. if so it is onto. otherwise, it is not onto.



Surjective Surjective Not Surjective

Ex:- Let $f(x) = x^2$, x is any real number and $f : R \to R$, then f is not onto. The reason is that we can find a real no. whose square is negative then Range of f can not be equal to R.

5. Bijective function :-

A function f from A to B is said to be Bijective if f is both injective and surjective i.e both one-to-one and onto.

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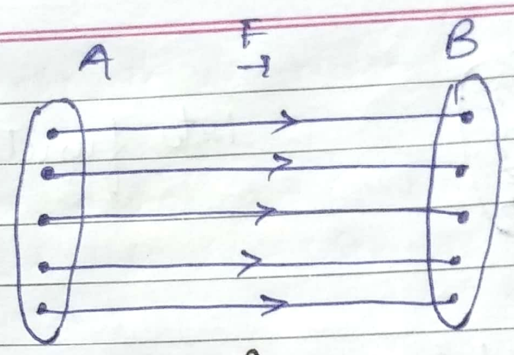
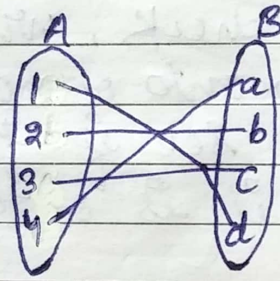


Bijective.

EX:- Let f be a function from A to B
$A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ with
$f(1) = d$, $f(2) = b$, $f(3) = c$, $f(4) = a$



then f is bijective function.
f is one-one since the function takes on different values. It is also onto since every element of B is the image of some element in A. Hence f is a bijective f^n.

Equal Functions :-

Two functions $f: A \to B$ and $g: X \to Y$ are said to be equal if $A = X$, $B = Y$, and

$$f(x) = g(x), \text{ for all } x \in A \ (= X)$$

Ques. Let A and B be two sets such that $|A| = |B|$. Prove that a function $f: A \to B$ is an injective function if and only if f is a surjective function.

Soln:-

In general, if $f: A \to B$ is Injective Then $|A| \leq |B|$. However, since $|A| = |B|$

$\Rightarrow$ Rang f $= B$

Thus f is a surjective function.

Conversely, if $f: A \to B$ is surjective we have Range f $= B$ so that $|A| \geq |B|$

However, since $|A| = |B|$, so no element $b \in B$ has more than one Pre-image.

Hence, f is an Injective function.

Ques. Let A and B be finite sets and $f: A \to B$ Then show that

i.) if f is one-to-one, then $|A| \leq |B|$

ii.) if f is onto, then $|B| \leq |A|$

iii.) if f is a bijection then $|A| = |B|$.

Solution:-

Let $A = \{a_1, a_2, \ldots, a_n\}$, $B = \{b_1, b_2, \ldots, b_m\}$

So that $|A| = n$ and $|B| = m$

i.) Suppose f is one-to-one function.

Then the n images of the elements of A

i.e $f(a_1), f(a_2), \ldots, f(a_n)$ are all different

Since all these n images belong to B,

So B Must have atleast n elements

i.e $|B| \geq n$ But $n = |A|$

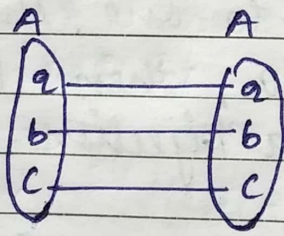
So $|B| \geq |A|$ i.e $|A| \leq |B|$. Hence Proved

ii) Suppose f is onto. Then every element of B is the image of some element in A. Since two or more elements (distinct) in A may have the same image in B, then $|B| \leq n$. i.e $|B| \leq |A|$.

iii) Suppose f is a bijection. Then f is both one-to-one and onto.
$\Rightarrow$ $|A| \leq |B|$ and $|B| \leq |A|$
Therefore $|A| = |B|$

Identity function :→

A function $f: A \to A$ defined by $f(x) = x$ for every $x \in A$ is called the Identity function of A. It is denoted by I_A.



Constant function :→

A function $f: R \to R$ defined by
$$f(x) = c \;,\; \forall\, x \in R$$
where $c \in R$ is a constant, is called Constant f^n.

Real Valued function :→

A function f from R to R i.e $f: R \to R$ is called Real valued f^n.

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Some Important functions :→

1.) **Floor Function :-**

An Integer-valued function $f: \mathbb{R} \to \mathbb{Z}$ given by

$$\lfloor x \rfloor = n, \text{ if } x \geq n, \quad x \in \mathbb{R}$$

is called the floor function.
It is also known as Greatest Integer funct^n.

e.g → $\lfloor 1.2 \rfloor = 1$, $\lfloor 0.5 \rfloor = 0$

2.) **Ceiling Function :-**

An Integer-valued function $f: \mathbb{R} \to \mathbb{Z}$ given by

$$\lceil x \rceil = n, \text{ where } x \leq n, \quad x \in \mathbb{R}$$

is called the ceiling Function.

e.g → $\lceil 1.2 \rceil = 2$, $\lceil 0.5 \rceil = 1$

Theorem :-

For any $x \in \mathbb{R}$

1.) $\lfloor x \rfloor = n \iff n \leq x < n+1$
$\lceil x \rceil = n \iff n-1 < x \leq n$

2.) $\lfloor x \rfloor = n \iff x-1 < n \leq x$
$\lceil x \rceil = n \iff x \leq n < x+1$

3.) $x-1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x+1$.

4.) $\lfloor -x \rfloor = -\lceil x \rceil$ and $\lceil -x \rceil = -\lfloor x \rfloor$

5.) $\lfloor x+n \rfloor = \lfloor x \rfloor + n$
$\lceil x+n \rceil = \lceil x \rceil + n$.

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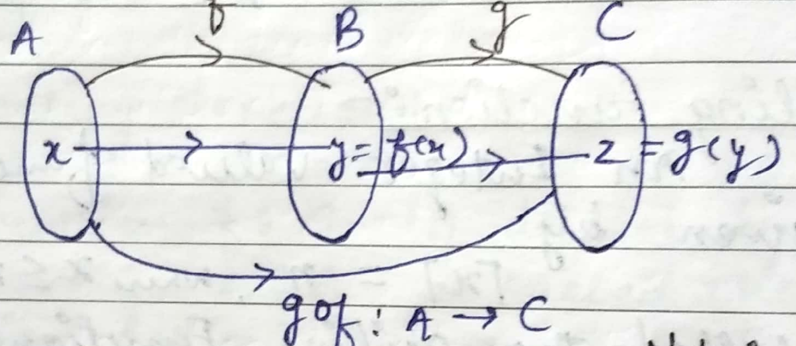
Composite function | Composition of Functions :-

Let A, B, and C be three non-empty sets and $f : A \to B$ & $g : B \to C$

Then the composition of f and g is a function denoted by $g o f : A \to C$ and is defined by

$$(g o f)(x) = g[f(x)] \quad \forall\ x \in A$$



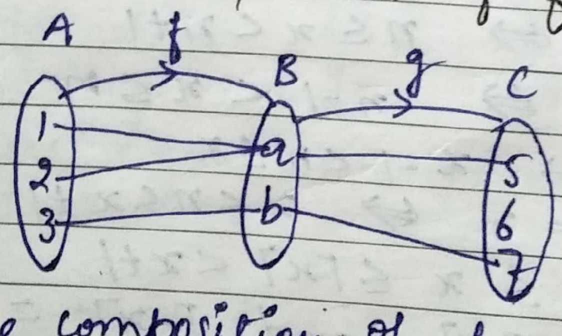
Note :- $f o g \neq g o f$

Ex :- 1 Let $A = \{1, 2, 3\}$, $B = \{a, b\}$, $C = \{5, 6, 7\}$

Define $f : A \to B$, $f = \{(1, a), (2, a), (3, b)\}$
and $g : B \to C$, $g = \{(a, 5), (b, 7)\}$

Find the composition of f and g.

Sol^n :-



The composition of f and g is $g o f$

$$(g o f)(x) = g[f(x)]$$

$$(g o f)(1) = g[f(1)] = g(a) = 5$$
$$(g o f)(2) = g[f(2)] = g(a) = 5$$
$$(g o f)(3) = g[f(3)] = g(b) = 7$$

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$\therefore$ $gof = \{(1,5), (2,5), (3,7)\}$

Ex:-2 Let $f: R \to R$ and $g: R \to R$ be two R.V. fn given by $f(x) = x^2$ and $g(x) = x+4$
Then
Find i.) gof ii.) fog iii.) fof iv.) gog

Soln:-
Given $f(x) = x^2$, $g(x) = x+4$

1.) $(gof)(x) = g[f(x)] = g(x^2)$
$= x^2+4$

2.) $(fog)(x) = f[g(x)] = f(x+4)$
$= (x+4)^2$

3.) $(fof)(x) = f[f(x)] = f(x^2)$
$= (x^2)^2$
$= x^4$

4.) $(gog)(x) = g[g(x)] = g(x+4)$
$= (x+4)+4$
$= x+8.$

Ex:-3 Find the composition of functions $f: N \to Z$ and $g: Z \to Z$ resp. given by $f(n) = n-2$ and $g(n) = n^2.$

Soln:- composition of f and g is
$(gof)(n)$
$= g[f(n)]$
$= g(n-2)$
$= (n-2)^2$
$= n^2-4n+4.$

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We can also find composition of g and f i.e $f \circ g$, However codomain $g \neq$ dom f

since Range$(g) = \{x^2 \mid x \in \mathbb{Z}\} \subseteq \mathbb{N}$

$\therefore$

$$(f \circ g)(x) = f[g(x)]$$
$$= f(x^2)$$
$$= x^2 - 2$$

Note:- $f \circ g \neq g \circ f$
because of two Reasons
i.) Their values f and g are different
ii.) the domain of $f \circ g$ and $g \circ f$ are not the same yet.

Ques. Consider the function $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ and $h: \mathbb{R}^* \to \mathbb{R}$ given by
$f(x) = 3x^2 + 2$, $g(x) = 7x - 5$,
$h(x) = \frac{1}{x}$.

Compute the composite functions $f \circ g \circ h$, $g \circ g$, $g \circ h$, $h \circ g \circ f$.

Solⁿ:-
i.) $(f \circ g \circ h)(x) = f \circ g[h(x)]$
$$= (f \circ g)\left(\frac{1}{x}\right)$$
$$= f\left[g\left(\frac{1}{x}\right)\right]$$
$$= f\left(\frac{7}{x} - 5\right)$$
$$= 3\left(\frac{7}{x} - 5\right)^2 + 2$$
$$= \frac{147}{x^2} + 77 - \frac{210}{x}$$

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ii.) $(gog)(x) = g[g(x)]$

$= g(7x-5)$

$= 7(7x-5)-5$

$= 49x - 35 - 5$

$= 49x - 40$

iii.) $(goh)(x) = g[h(x)]$

$= g[\frac{1}{x}]$

$= \frac{7}{x} - 5$

iv.) $(goh)(x) = g[h(x)]$

v.) $(hogof)(x) = hog[f(x)]$

$= (hog)(3x^2+2)$

$= h[g(3x^2+2)]$

$= h(7(3x^2+2)-5)$

$= h(21x^2+9)$

$= \frac{1}{21x^2+9}$

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Th^m- Let $f: A \to B$ and $g: B \to C$ be two functions such that the composite function $gof: A \to C$ is a bijective function. Then f is injective and g is surjective function.

Proof:-

Given that the composite function $gof: A \to C$ is a bijective function i.e one-one and onto

Since gof is one-one / Injective

So for $x_1, x_2 \in A$,

$$gof(x_1) = gof(x_2) \Rightarrow x_1 = x_2 \quad \text{—} ①$$

Now to show $f: A \to B$ is one-one we have to show that

for $x_1, x_2 \in A$, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$f(x_1) = f(x_2)$

taking g function on both the sides

$gof(x_1) = gof(x_2)$

$\Rightarrow \quad x_1 = x_2 \quad$ from(1)

Hence f is one-one / Injective function.

Since gof is onto fn

So for all $z \in C$, $\exists$ $x \in A$ such that

$$gof(x) = z$$

Now to show $g: B \to C$ is onto we have to show that

for all $z \in C$, $\exists$ $y \in B$ such that

$$g(y) = z$$

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let $x \in A$, $y \in B$ and $z \in C$
since gof is onto
$\Rightarrow \exists\ x \in A$ such that $gof(x) = z$
$\Rightarrow g(f(x)) = z$
$\Rightarrow g(y) = z \qquad \because y = f(x)$

$\Rightarrow$ for every $z \in C$, $\exists\ y \in B$ such that
$$g(y) = z$$

Hence g is onto / Surjective f^n.

Imp
Theorem:- Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be one-one and onto functions. Then the composite function $gof: A \rightarrow C$ is also one-one and onto function and
$$(gof)^{-1} = f^{-1} o g^{-1}$$

Proof:-
Since f is one-one, for $x_1, x_2 \in A$
$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$
again g is one-one, for $y_1, y_2 \in B$
$$g(y_1) = g(y_2) \Rightarrow y_1 = y_2$$

Now $(gof)(x_1) = (gof)(x_2)$
$\Rightarrow g[f(x_1)] = g[f(x_2)]$

$\Rightarrow f(x_1) = f(x_2) \qquad$ Since g is one-one
$\Rightarrow x_1 = x_2 \qquad$ since f is one-one

Therefore $(gof)(x_1) = (gof)(x_2) \Rightarrow x_1 = x_2$

Hence gof is one-one function.

Since g is onto, So for each $z \in C$, $\exists\, y \in B$ such that $g(y) = z$

Also, f is onto, So for each $y \in B$, $\exists\, x \in A$ such that $f(x) = y$

Now, for each $z \in C$,

$$gof(x) = g[f(y)]$$
$$= g(y)$$
$$= z$$

So $\forall\, z \in C$, $\exists\, x \in A$ s.t $gof(x) = z$

$\Rightarrow$ every element $z \in C$ has preimage under gof

Hence gof is an onto function.

Now gof is one-one and onto f^n

So $(gof)^{-1}$ exists

By the def. of composite functions, if

$gof : A \to C$, so $(gof)^{-1} : C \to A$

Also $f^{-1} : B \to A$ and $g^{-1} : C \to B$

Then by def. of composite f^ns

$f^{-1}og^{-1} : C \to A$

Therefore, domain of $(gof)^{-1}$ = domain of $f^{-1}og^{-1}$

Now $(gof)^{-1}(z) = x$

$\Leftrightarrow (gof)(x) = z$

$\Leftrightarrow g[f(x)] = z$

$\Leftrightarrow g(y) = z$ where $y = f(x)$

$\Leftrightarrow y = g^{-1}(z)$

$\Leftrightarrow f^{-1}(y) = f^{-1}[g^{-1}(z)] = (f^{-1}og^{-1})(z)$

$\Leftrightarrow x = (f^{-1}og^{-1})(z)$

Thus $(gof)^{-1}(z) = (f^{-1}og^{-1})(z)$, So $(gof)^{-1} = f^{-1}og^{-1}$

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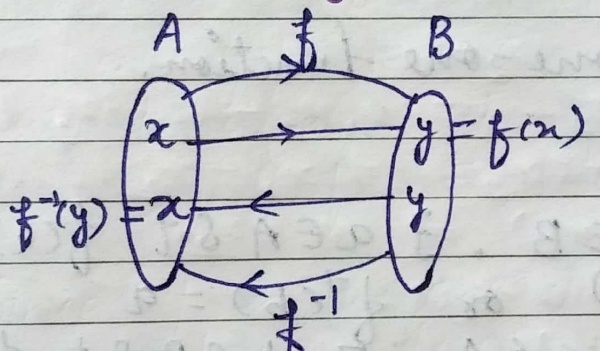
Inverse of a function :—

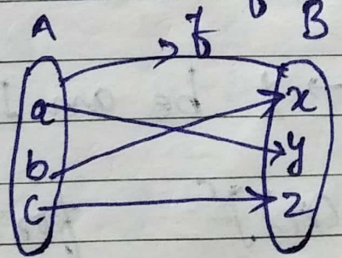
Let $f : A \to B$ be a function. Then $g : B \to A$ is called the inverse of f if
$g \circ f = I_A$ and $f \circ g = I_B$
i.e $g[f(x)] = x, \forall x \in A$ and $f[g(y)] = y, \forall y \in B$

It is denoted by f^{-1}.
Thus $f(x) = y \iff x = f^{-1}(y)$.

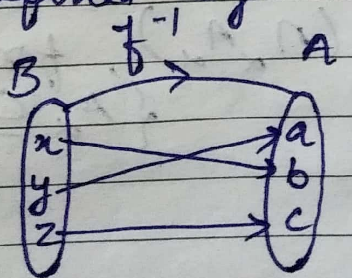
Note:- The Inverse of $f : A \to B$ exists if and only if f is bijective i.e one-one and onto f".



Ex:- Let function $f : A \to B$ be defined by



Then f is one-one and onto function
Therefore, f^{-1} exists and $f^{-1} : B \to A$ is defined by



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Theorem:- If a mapping $f: A \to B$ is one-one and onto, then the Inverse mapping $f^{-1}: B \to A$ is also one-one and onto

Proof:-

$f: A \to B$ is one-one and onto $\Rightarrow f^{-1}$ exists

Let $a_1, a_2 \in A$ and $b_1, b_2 \in B$ so that

$f(a_1) = b_1$ and $f(a_2) = b_2$

$\Rightarrow a_1 = f^{-1}(b_1)$ and $a_2 = f^{-1}(b_2)$

Now $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$ ($\because f$ is 1-1)

or $b_1 = b_2 \Leftrightarrow f^{-1}(b_1) = f^{-1}(b_2)$

i.e $f^{-1}(b_1) = f^{-1}(b_2) \Rightarrow b_1 = b_2$

$\therefore f^{-1}$ is one-one function.

As f is onto

$\Rightarrow$ for each $b \in B$, $\exists\, a \in A$ s.t $f(a) = b$

$\Rightarrow a = f^{-1}(b)$ or $f^{-1}(b) = a$

So for any $a \in A$, $\exists\, b \in B$ s.t $f^{-1}(b) = a$

Hence f^{-1} is onto.

Theorem:- Let $f: X \to Y$ be an Invertible f^n Then

1. $f^{-1}(A^c) = [f^{-1}(A)]^c$ for all $A \subseteq X$
2. $\cup_\alpha f^{-1}(A_\alpha) = f^{-1}[\cup_\alpha A_\alpha]$, for every class $\{A_\alpha \subseteq X : \alpha \in I\}$
3. $\cap_\alpha f^{-1}(A_\alpha) = f^{-1}(\cap_\alpha A_\alpha)$, for every class $\{A_\alpha \subseteq X : \alpha \in I\}$

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Ex:- Let $f: R \to R$ is defined by
$f(x) = ax + b$ where $a, b, x \in R$ & $a \neq 0$
Show that f is invertible and find the Inverse of f.

Sol:-

A function is Invertible iff it is one-one and onto

Given $f: R \to R$ s.t $f(x) = ax + b$, $a, b, x \in R, a \neq 0$ —— (1)

One-one :→ Let $x_1, x_2 \in R$

$f(x_1) = f(x_2)$

$\Rightarrow ax_1 + \cancel{b} = ax_2 + \cancel{b}$

$\Rightarrow ax_1 = ax_2$ $\because a \neq 0$

$\Rightarrow x_1 = x_2$

So $f(x_1) = f(x_1) \Rightarrow x_1 = x_2$

Hence f is one-one

Onto :→ $\forall\, y \in R$, $\exists$ an element $x \in R$ Such that $f(x) = y$

$\Rightarrow ax + b = y$ from (1)

$\Rightarrow ax = y - b$

$\Rightarrow x = \dfrac{y-b}{a} \in R$ —— (2)

$\therefore \forall\, y \in R$ $\exists$ an element $x = \dfrac{y-b}{a} \in R$

Such that $f(x) = f\left(\dfrac{y-b}{a}\right)$

$= a\left(\dfrac{y-b}{a}\right) + b$

$= y$

$\Rightarrow f(x) = y$

Hence f is onto function.

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To find f^{-1}:- $x = f^{-1}(y)$ from (2)

$$f^{-1}(y) = \frac{y-b}{a}$$

Hence $\boxed{f^{-1}(y) = \frac{y-b}{a}}$

Imp (2022)

Ques. Let a fn $f: R-\{3\} \to R-\{1\}$ be defined as $f(x) = \frac{x-1}{x-3}$. Show that f is bijective fn and find f^{-1}.

Soln:- To show that f is one-one,
let $x_1, x_2 \in R-\{3\}$ so that

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1-1}{x_1-3} = \frac{x_2-1}{x_2-3}$$

$$\Rightarrow x_1x_2 - 3x_1 - x_2 + 3 = x_1x_2 - x_1 - 3x_2 + 3$$

$$\Rightarrow -2x_1 = -2x_2 \Rightarrow x_1 = x_2$$

So $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
Therefore f is one-one.

Now $y = f(x)$

$$\Rightarrow y = \frac{x-1}{x-3}$$

$$\Leftrightarrow x = \frac{3y-1}{y-1}$$

So $\forall\, y \in R-\{1\}$, $\exists\, x = \frac{3y-1}{y-1} \in R-\{3\}$ s.t $f(x) = y$
$\therefore f$ is onto.

Therefore, $g: R-\{1\} \to R-\{3\}$ given by $g(y) = \frac{3y-1}{y-1}$ is a well defined bijective function such that

$$fog = I_{R-\{1\}} \text{ and } gof = I_{R-\{3\}}$$

Hence g is the Inverse of f.

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Growth of Functions :→

The Growth of functions is described using three Important Notations :

1.) Big-oh (O)
2.) Big-omega (Ω)
3.) Big-theta (θ)

They Provide a special way to compare relative size of functions that is very useful in analysis of computer algorithm.

suppose M is an algorithm and n is the size of the Input data. clearly the complexity $f(n)$ of M increases as n increases. If we know how the function $f(n)$ grows, we can determine the efficiency of the algorithm and this done by comparing $f(n)$ with some standard functions.

$$1 < \log n < \sqrt{n} < n < n \log n < n^2 < n^3 \ldots < 2^n < 3^n < \ldots n^n$$

Imp

1.) Big-Oh Notation:→

Let $f(x)$ and $g(x)$ be two Real valued functions, where the argument x takes a real value or it is non-negative Integer. Then $f(x)$ is of order g i.e $f(x) = O(g(x))$ if there are positive constants C and k such that

$$|f(x)| \le C|g(x)| \quad , \text{ for all } x > k$$

The two constants C and α are called witness

Equivalently, if $\lim_{x \to \infty} [f(x) / g(x)] < \infty$.

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Big-omega and Big-theta Notations | Big-Ω & Big-Θ

Big-Ω →

Let $f(x)$ and $g(x)$ be two functions from the set of integers or the set of real numbers to the set of real numbers. Then $f(x) = \Omega(g(x))$ if there are positive constants C and k such that

$$|f(x)| \geq C|g(x)| \quad \text{whenever } x > k$$

which is read as "$f(x)$ is big-omega of $g(x)$"

Big-Θ →

Let $f(x)$ and $g(x)$ be two functions from the set of Integers or the set of Real numbers to the set of real numbers. Then $f(x)$ is $\Theta\, g(x)$ if $f(x)$ is $O\, g(x)$ and $f(x)$ is $\Omega\, g(x)$.

When $f(x)$ is $\Theta\, g(x)$, we say that "f is big-theta of $g(x)$" and denoted by $f(x) = \Theta[g(x)]$

Note:-

If $f(x)$ is $O\, g(x)$, then $g(x)$ is an upper bound for $f(x)$ and whenever $f(x)$ is $\Omega\, g(x)$, then $g(x)$ is a lower bound for $f(x)$.

The Big-O Notation compares the rate of growth of functions rather than their values.

So when $f(x)$ is $\Theta\, g(x)$, $f(x)$ and $g(x)$ have the same rates of growth, but can be very different in their values.

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Boolean Algebra :→

A non-empty set B with two binary operations $+$ and $\cdot$, a unary operation $'$, and two distinct elements 0 and 1 is called a Boolean Algebra, denoted by $(B, +, \cdot, ', 0, 1)$ iff the following properties are satisfied, for any $a, b, c \in B$

1. Commutative laws :→

   (a) $a + b = b + a$ (b) $a \cdot b = b \cdot a$.

2. Distributive laws :→

   (a) $a + b \cdot c = (a + b) \cdot (a + c)$

   (b) $a \cdot (b + c) = a \cdot b + a \cdot c$

3. Identity laws :→

   (a) $a + 0 = a$ (b) $a \cdot 1 = a$

4. Complement laws :→

   (a) $a + a' = 1$ (b) $a \cdot a' = 0$.

Some Basic Theorems

\# Let $(B, +, \cdot, ', 0, 1)$ be a Boolean Algebra and $a, b, c \in B$ then

1. Idempotent laws :

   (a) $a + a = a$ (b) $a \cdot a = a$.

2. Boundedness laws :

   (a) $a + 1 = 1$ (b) $a \cdot 0 = 0$

3. Absorption laws :

   (a) $a + (a \cdot b) = a$ (b) $a \cdot (a + b) = a$

4. Associative laws :

   (a) $(a + b) + c = a + (b + c)$

   (b) $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

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5. Uniqueness of complement:
$a + x = 1$ and $a \cdot x = 0$, then $x = a'$

6. Involution law:
$(a')' = a$

7. ⓐ $0' = 1$ ⓑ $1' = 0$

8. De morgan's law:
ⓐ $(a+b)' = a' \cdot b'$ ⓑ $(a \cdot b)' = a' + b'$

Proofs :-

(1) ⓐ $a + a = a$

$a = a + 0$ by Identity law
$= a + a \cdot a'$ using Complement law
$= (a+a) \cdot (a+a')$ using Distributive law
$= (a+a) \cdot 1$ using complement law
$= a + a$ Using Identity law.

ⓑ $a \cdot a = a$

$a \cdot a = a \cdot a + 0$ by Identity law
$= a \cdot a + a \cdot a'$ using complement law
$= a \cdot (a + a')$ using distributive law
$= a \cdot 1$ using complement law
$= a$ using Identity law.

(2) ⓐ $a + 1 = 1$

$a + 1 = a + (a + a')$ by complement law
$= (a+a) + a'$ using Associative law
$= a + a'$ using Idempotent law
$= 1$ using Identity law.

ⓑ $a \cdot 0 = 0$

Ily $a \cdot 0 = a \cdot (a \cdot a') = (a \cdot a) \cdot a'$
$= a \cdot a' = 0$ using complement, Associative, Idempotent laws

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(3) (a) $a + a\cdot b = a$

$a + (a\cdot b) = a\cdot 1 + a\cdot b$ by Identity law

$= a\cdot(1+b)$ using Distributive law

$= a\cdot 1$ using Boundedness law

$= a$ using identity law

~~(b) $a\cdot(a+b) = a$~~

~~$a\cdot(a+b) = a\cdot a + a\cdot b$ by distributive law~~

~~$= a + a\cdot b$ by Idempotent law~~

~~$= a$ from (a)~~

(b) $a\cdot(a+b) = a$

$a\cdot(a+b) = (a+0)\cdot(a+b)$ by Identity law $a+0=a$

$= a + 0\cdot b$ by distributive law

$= a + 0$ by Boundedness law

$= a$ by identity law.

Imp

Thm In a Boolean Algebra B, the identity elements 0 and 1 are complementary to each other.

OR

for $0, 1 \in B$, we have (i) $0' = 1$ (ii) $1' = 0$.

Proof:- (i) $0' = 1$

$0' = 0' + 0$ [by Identity law $a + 0 = a$]

$= 1$ [by complement law $a + a' = 1$]

(ii) $1' = 0$

$1' = 1'\cdot 1$ [by Identity law $a\cdot 1 = a$]

$= 0$ [by complement law $a\cdot a' = 0$]

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Theorem 5: for each $a \in B$, a' is unique.

proof:-

Let, if possible, a' and a'' be two complements of a.

Then $a + a' = a' + a = 1$ & $a \cdot a' = a' \cdot a = 0$

$a + a'' = a'' + a = 1$ & $a \cdot a'' = a'' \cdot a = 0$

Now

$$
\begin{aligned}
a' &= a' \cdot 1 \quad \text{(Identity law)} \\
&= a' \cdot (a + a'') \\
&= a' \cdot a + a' \cdot a'' \\
&= 0 + a' \cdot a'' \\
&= a \cdot a'' + a' \cdot a'' \\
&= (a + a') \cdot a'' \\
&= 1 \cdot a'' \\
&= a''
\end{aligned}
$$

So $a' = a''$

Hence the complement a' of $a \in B$ is unique.

Theorem:- The elements 0 and 1 in a Boolean Algebra are unique.

Proof:- Suppose that there are two zero elements 0_1 and 0_2. Then for each element $a \in B$ and $b \in B$, we have

$a + 0_1 = a$ and $b + 0_2 = b$ (identity law)

Let $a = 0_2$ and $b = 0_1$

Thus $0_2 + 0_1 = 0_2$ and $0_1 + 0_2 = 0_1$ —(1)

But $0_2 + 0_1 = 0_1 + 0_2$ (Commutative law)

from (1)

$$\boxed{0_2 = 0_1}$$

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thus the zero element is unique.

Similarly, let there are two identity elements 1_a and 1_b then for each $c \in B$ and $d \in B$ we have,

$c \cdot 1_a = c$ and $d \cdot 1_b = d$ (Identity law)

Let $c = 1_b$ and $d = 1_a$

Thus $1_b \cdot 1_a = 1_b$ and $1_a \cdot 1_b = 1_a$ — (11)

But $1_b \cdot 1_a = 1_a \cdot 1_b$ (commutative law)

from (11),

$$\boxed{1_b = 1_a}$$

Thus, the Identity element 1 is unique.

Theorem:- De-morgan's laws:

for any $a, b \in (B, +, \cdot, ', 0, 1)$

(i) $(a+b)' = a' \cdot b'$

(ii) $(a \cdot b)' = a' + b'$.

Proof:-

Theorem:-6: for each element a of Boolean Algebra B, $(a')' = a$.

proof:- Let a' be the complement of $a \in B$

Then $a' \in B$

$\Rightarrow$ $a + a' = 1$ and $a \cdot a' = 0$ — (1)

But $a + a' = a' + a$ and $a \cdot a' = a' \cdot a$ (comm. law)

$\Rightarrow$ $a' + a = 1$ and $a' \cdot a = 0$ from (1) — (2)

eqn (2) shows that complement of $a' \in B$ is $a \in B$

But complement of each element of Boolean algebra is unique. Hence, $\boxed{(a')' = a}$

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Theorem 1.8 De-Morgan's laws:

for any $a, b \in B$, we have

(i) $(a+b)' = a' \cdot b'$

(ii) $(a \cdot b)' = a' + b'$

Proof:-

(i) To prove that $(a+b)' = a' \cdot b'$

we have to show that the complement of $(a+b)$ is $a' \cdot b'$ and for that we have to show that

$$(a+b) + a' \cdot b' = 1 \text{ and } (a+b) \cdot (a' \cdot b') = 0$$

Now

$(a+b) + a' \cdot b' = [(a+b) + a'] \cdot [(a+b) + b']$ (by distributive law)

$= [a' + (a+b)] \cdot [a + (b+b')]$ using commutative & Associative law

$= [(a'+a)+b] \cdot [a+(b+b')]$ "

$= (1+b) \cdot (a+1)$ using complement law

$= 1 \cdot 1$ using Boundedness law

$= 1$ using Identity law.

and $(a+b) \cdot (a'b') = a \cdot (a'b') + b \cdot (a'b')$ by distributive law

$= (aa')b' + b(b'a')$ using Associative & commutative law

$= (aa')b' + (bb')a'$ "

$= 0 \cdot b' + 0 \cdot a'$ (Complement law)

$= 0 + 0$ (Boundedness law)

$= 0$ Idempotent law

Hence

$$\boxed{(a+b)' = a' \cdot b'}$$

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(ii) To show that the complement of $(a.b)$ is $a'+b'$ we have to show that

$(a.b)+(a'+b')=1$ and $(a.b).(a'+b')=0$

Now $(a.b)+(a'+b') = [a+(a'+b')].[b+(a'+b')]$ By distrutive law.

$=[(a+a')+b'].[b+(b'+a')]$ by Associative & commutative law

$=[(a+a')+b'].[(b+b')+a']$ by Associative law

$=(1+b').(1+a')$ by complement law

$=1.1$ by Boundedness law

$=1$ by Identity law.

and $(a.b).(a'+b') = (a.b).a'+(a.b).b'$ By distributive law.

$=(b.a).a'+(a.b)b'$ by commutative law

$=b(a.a')+a.(b.b')$ by Associative law

$=b.0+a.0$ by complement law

$=0+0$ by Boundedness law

$=0$ by Idempotent law.

Hence $\boxed{(a.b)'=a'+b'}$

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(#) Boolean SubAlgebra :-

Example :- 1 Let $B = \{1, 5, 7, 35\}$ be the set of the Integers and operations + and · are defined as

$$a + b = lcm(a, b)$$

and $a \cdot b = gcd(a, b) \quad \forall a, b \in B$

A unary operation ' on B is defined as

$$a' = 35/a \quad \forall a \in B$$

Show that $(B, +, \cdot, ')$ is a Boolean algebra.

Solution:-

Here $B = \{1, 5, 7, 35\}$.

The composition table for the binary operations + and · are given as

+	1	5	7	35
1	1	5	7	35
5	5	5	35	35
7	7	35	7	35
35	35	35	35	35

·	1	5	7	35
1	1	1	1	1
5	1	5	1	5
7	1	1	7	7
35	1	5	7	35

closure law!

(1) It is clear from the table that + and · are binary operations on B

i.e $a + b, a \cdot b \in B, \forall a, b \in B$

(2) commutative law! Since all rows and coloumns in both the composition tables are identical Therefore, the commutative law hold for + &.

i.e (i) $a + b = b + a$ & (ii) $a \cdot b = b \cdot a \quad \forall a, b \in B$

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(3) Distributive law :— $\forall a, b, c \in B$

$$a + (b \cdot c) = (a+b) \cdot (a+c)$$

$$a \cdot (b+c) = a \cdot b + a \cdot c$$

Hence the Distributive law holds.

for $a = 7,\ b = 5,\ c = 1$.

$$a + (b \cdot c) = 7 + (5 \cdot 1) = 7 + 1 = 7$$

~~$a \cdot (b+c) = 7 \cdot (5+1) = 7 \cdot 5 =$~~

$$(a+b) \cdot (a+c) = (7+5) \cdot (7+1) = 35 \cdot 7 = 7$$

$\Rightarrow$ $a + (b \cdot c) = (a+b) \cdot (a+c)$

and $a \cdot (b+c) = 7 \cdot (5+1) = 7 \cdot 5 = 1$

$$(a \cdot b) + (a \cdot c) = (7 \cdot 5) + (7 \cdot 1) = 1 + 1 = 1$$

$\Rightarrow$ $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$.

(4) Identity law :— here 1 and 35 are least & greatest element respectively.

a	1	$a+1$
1	1	1
5	1	5
7	1	7
35	1	35

a	35	$a \cdot 35$
1	35	1
5	35	5
7	35	7
35	35	35

Since $a + 1 = a, \ \forall a \in B$

& $a \cdot 35 = a, \ \forall a \in B$.

Hence identity law holds. Here 1 is the zero element and 35 is the unit element.

(5) Complement law :—

given $a' = 35/a \quad \forall a \in B$

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a	a'	a+a'	a.a'
1	35	35	1
5	7	35	1
7	5	35	1
35	1	35	1

$\therefore a + a' = 35 \quad \forall a \in B$

$\& \quad a \cdot a' = 1 \quad \forall a \in B$

Hence complement law holds

$\therefore$ $(B, +, \cdot, ')$ is a Boolean Algebra.

Imp

Example: 2 Show that the set $B = \{1, 2, 3, 6\} = D_6$ is a Boolean Algebra with respect to binary operations + and · and unary operation respectively given by

$$a + b = \text{lcm}\{a, b\}$$
$$a \cdot b = \text{hcf}\{a, b\}$$
$$a' = 6/a \quad \forall a, b \in B.$$

Solⁿ:-

Here $B = \{1, 2, 3, 6\}$

The composition table for the binary operation + and · are as follows

+	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

·	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

(1) closure law:- Since all the entries are from the set B in both tables

i.e $a+b, a \cdot b \in B, \quad \forall a, b \in B.$

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Therefore, closure law holds.

(2) Commutative law :— Since all the corresponding Rows and coloumns are Identical in both composition tables
i.e $a+b = b+a$ & $a \cdot b = b \cdot a \ \forall a,b \in B$.
$\Rightarrow$ commutative law holds.

(3) Distributive law :— $\forall a,b \in B$
$$a+(b \cdot c) = (a+b) \cdot (a+c)$$
& $$a \cdot (b+c) = (a \cdot b) + (a \cdot c)$$
Hence Distributive law holds.
for $a=3$, $b=2$ & $c=1$, we have
$$a+(b \cdot c) = 3+(2 \cdot 1) = 3+1 = 3$$
$$(a+b) \cdot (a+c) = (3+2) \cdot (3+1) = 6 \cdot 3 = 3$$
$$\Rightarrow a+(b \cdot c) = (a+b) \cdot (a+c)$$
and $$a \cdot (b+c) = 3 \cdot (2+1) = 3 \cdot 2 = 1$$
$$(a \cdot b)+(a \cdot c) = 3 \cdot 2 + 3 \cdot 1 = 1+1 = 1$$
$$\Rightarrow a \cdot (b+c) = (a \cdot b)+(a \cdot c).$$

(4) Identity law :— Here 1 and 6 are zero and identity elements respectively.
(least) (Greatest)

a	1	a+1
1	1	1
2	1	2
3	1	3
6	1	6

a	6	a·6
1	6	1
2	6	2
3	6	3
6	6	6

Since $a+1 = a$
& $a \cdot 6 = a$ $\forall a \in B$
Hence the Identity law holds

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(5.) Complement law :—
we have $a' = 6/a \quad \forall a \in B$

a	a'	$a+a'$	$a \cdot a'$
1	6	6	1
2	3	6	1
3	2	6	1
6	6	6	1

Since $a + a' = 6$
and $a \cdot a' = 1 \quad \forall a \in B$

$\Rightarrow$ complement law holds

Hence $(B = \{1, 2, 3, 6\}, +, \cdot, ')$ is a Boolean Algebra.

Ex:-3 Show that the set $B = \{1, 2, 4, 8\}$ is not a Boolean Algebra with respect to the binary operations $+$, $\cdot$ and unary operation $'$ respectively given by

$$a + b = \text{lcm}\{a, b\}$$
$$a \cdot b = \text{hcf}\{a, b\}$$
and $$a' = 8/a$$

Solution:- Here $B = \{1, 2, 4, 8\}$.
The composition tables for the binary operations $+$ and $\cdot$ are as

$+$	1	2	4	8
1	1	2	4	8
2	2	2	4	8
4	4	4	4	8
8	8	8	8	8

$\cdot$	1	2	4	8
1	1	1	1	1
2	1	2	2	2
4	1	2	4	4
8	1	2	4	8

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(1) Closure law :- Since all the entries in both composition tables are coming from B

i.e $a+b,\ a\cdot b \in B,\ \forall\ a,b \in B$

Hence closure law holds.

(2) Commutative law :- Since all corresponding rows & columns are identical in both the tables

Hence commutative law holds

i.e $a+b = b+a$ & $a\cdot b = b\cdot a,\ \forall a,b \in B$

(3) Distributive law :- for $a=4, b=2, c=1$

$a+(b\cdot c) = 4+(2\cdot 1) = 4+1 = 4$

$(a+b)\cdot(a+c) = (4+2)\cdot(4+1) = 4\cdot 4 = 4$

for $a=8,\ b=4,\ c=2.$

$a+(b\cdot c) = 8+(4\cdot 2) = 8+2 = 8$

$(a+b)\cdot(a+c) = (8+4)\cdot(8+2) = 8\cdot 8 = 8$

for $a=8,\ b=2,\ c=1$

$8+(2\cdot 1) = 8+1 = 8$

$(8+2)\cdot(8+1) = 8\cdot 8 = 8$

$\therefore\ a+(b\cdot c) = (a+b)\cdot(a+c)\ \forall a,b,c \in B$

Similarly $a\cdot(b+c) = (a\cdot b)+(a\cdot c)\ \forall a,b,c \in B$

Hence Distributive law holds.

(4.) Identity law :- Here 1 and 8 are zero (least) and unit (greatest) element

a	1	a+1
1	1	1
2	1	2
4	1	4
8	1	8

a	8	a·8
1	8	1
2	8	2
4	8	4
8	8	8

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Since $a + 1 = a$
& $a \cdot 8 = a$ $\forall\, a \in B$

Hence Identity law holds.

(8.) Commutative law:—

Given $a' = 8/a$ ~~$a \in B$~~

a	a'	$a+a'$	$a \cdot a'$
1	8	8	1
2	4	8	1
4	2	8	1
8	1	8	1

$\because 2+4 = 4 \neq 8$
& $4+2 = 4 \neq 8$
and $2 \cdot 4 = 2 \neq 1$
$4 \cdot 2 = 2 \neq 1$.

Since $\nexists$ any $a \in B$ s.t
$2 + a = 8$ & $2 \cdot a = 1$

Similarly $\nexists$ any $b \in B$ s.t
$4 + b = 8$ and $4 \cdot b = 1$

i.e The complements of 2 and 4 does not exist

Therefore, The complement law does not hold

Hence $(B = \{1, 2, 4, 8\}, +, \cdot, ')$ is not a Boolean Algebra.

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Boolean SubAlgebra :→

A non-empty Subset A of a Boolean Algebra $(B, +, \cdot, ', 0, 1)$ is called a Boolean Subalgebra of A is a Boolean Algebra under the same set of binary and unary operations i.e
$a, b \in A \Rightarrow a+b, a \cdot b, a' \in A$.

Ex:- consider the set of all +ve Divisors of 30
i.e $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and let $+$ & $\cdot$ & $'$ be defined as

$$a+b = \text{lcm}(a, b)$$
$$a \cdot b = \gcd(a, b) \text{ and } a' = 30/a.$$

Then D_{30} is a Boolean Algebra and $A_1 = \{1, 5, 6, 30\}$, $A_2 = \{1, 3, 10, 30\}$ & $A_3 = \{1, 5, 6, 30\}$ are Boolean SubAlgebra of D_{30}.

Sol^n:- $A_1 = \{1, 5, 6, 30\}$.

The composition table for binary operat^n + & . & unary operation ' are as follows

+	1	5	6	30
1	1	5	6	30
5	5	5	30	30
6	6	30	6	30
30	30	30	30	30

·	1	5	6	30
1	1	1	1	1
5	1	5	1	5
6	1	1	6	6
30	1	5	6	30

a	a'
1	30
5	6
6	5
30	1

Since $a+b, a \cdot b, a' \in A$, $\forall a, b \in A$,
Hence A_1 is a Subalgebra of B.

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$A_2 = \{1, 3, 10, 30\}$

+	1	3	10	30
1	1	3	10	30
3	3	3	30	30
10	10	30	10	30
30	30	30	30	30

·	1	3	10	30
1	1	1	1	1
3	1	3	1	3
10	1	1	10	10
30	1	3	10	30

a	a'
1	30
3	10
10	3
30	1

$\therefore$ $a+b$, $a.b$ & $a' \in A_2$, $\forall a, b \in A_2$

Hence A_2 is a Subalgebra of B.

$A_3 = \{1, 5, 6, 30\}$

+	1	5	6	30
1	1	5	6	30
5	5	5	30	30
6	6	30	6	30
30	30	30	30	30

·	1	5	6	30
1	1	1	1	1
5	1	5	1	5
6	1	1	6	6
30	1	5	6	30

a	a'
1	30
5	6
6	5
30	1

Since $a+b$, $a.b$, $a' \in A_3$, $\forall a, b \in A_3$

Hence A_3 is a Subalgebra of B.

Boolean function or Boolean Polynomial :→

An Expression obtained By the application of binary operations (+, ·) and uniary operation (') on finite no. of elements of Boolean Algebra (B, +, ·, ') is called a Boolean function or Boolean ~~Algebra~~. Polynomial.

Example :-

(1.) $f(x) = x + x'$

(2) $f(x, y) = x \cdot y' + x' \cdot y$

(3) $f(x, y, z) = x \cdot y \cdot z + x \cdot y' \cdot z + x' \cdot y' \cdot z$

Minterm :→

A minterm of n variables $x_1, x_2, -- x_n$ is a product of n literals in which each variable appears exactly once in either true or complimented form (i.e x_i or x_i'), but not both.

e.g →

The minterms of two variables x and y are $xy, x'y, xy', x'y'$.

The minterms of three variables x, y, z are $xyz, x'yz, xy'z, xyz', x'y'z, xy'z', x'yz', x'y'z'$

Note :- (1.) The No. of minterms in n-variables are 2^n.

(2) The output ~~of~~ value of a minterm is 1.

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Maxterm : →

A maxterm of n-variables $x_1, x_2, \ldots x_n$ is sum of n - literals in which each variable appears exactly once in either true or complemented form, but not both.

e.g →

The maxterms of two variables x and y are
$x+y$, $x'+y$, $x+y'$, $x'+y'$.

The maxterms of three variables x, y, z are
$x+y+z$, $x'+y+z$, $x+y'+z$, $x+y+z'$,
$x'+y'+z$, $x'+y+z'$, $x+y'+z'$, $x'+y'+z'$.

Note :-

1. The No. of maxterms in n-variables are 2^n.
2. The output value of maxterm is always 1.

Disjunctive Normal form (DNF) OR Sum of Product (SOP) : →

A Boolean function f which is written as a sum of minterms is called disjunctive normal form or DN form, or SOP or Canonical form

Examples :-

1. $f(x,y) = xy + x'y$
2. $f(x,y,z) = x'y'z + x'yz' + xy'z' + xyz$

⇒ **complete disjunctive Normal form : →**

A Boolean function f which is written as a sum of all 2^n minterms of n - variables is called complete disjunctive normal form

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eg – $f(x, y) = xy + xy' + x'y + x'y'$.

Example: Express the following function into DN form
$$f(x, y) = x + x'.y$$

Sol^n – No. of minterms in 2 variables $= 2^2 = 4$

Truth Table method

x	y	x'	$x'.y$	$x + x'.y$
1	1	0	0	1
1	0	0	0	1
0	1	1	1	1
0	0	1	0	0

$1 \rightarrow x$ or y
$0 \rightarrow x'$ or y'

Minterms present – $\underset{m_1}{xy}, \underset{m_2}{xy'}, \underset{m_3}{x'y}$

Therefore the Required DN form or SOP is

$$f(x, y) = m_1 + m_2 + m_3$$
$$= xy + xy' + x'y.$$

By Algebraic method:
$$f(x, y) = x + x'.y$$

$$f(x, y) = x + x'.y$$
$$= x.1 + x'.y \quad (\because a.1 = a \text{ identity law})$$
$$= x.(y + y') + x'.y \quad (\because a + a' = 1)$$
$$= x.y + x.y' + x'.y \quad [\text{Distributive law } a.(b+c) = a.b + a.c]$$

This is the Required DN form.

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Ques1. Express the Boolean function $f(x,y,z) = x + y'z$ in DN form or Sum of minterms

Sol^n. – $f(x,y,z) = x + y'z$ $(\because a + a' = 1)$

$= x.(y+y') + y'z$

$= xy + xy' + y'z$

$= xy(z+z') + xy'(z+z') + y'z(x+x')$

$= xyz + xyz' + \underline{xy'z} + xy'z' + \underline{xy'z} + x'y'z$

$(\because a + a = a)$

$= xyz + xyz' + xy'z + xy'z' + x'y'z$

This is the Required DN form.

Q.2. Find the Sum of product of the Boolean f^n $f(x,y,z) = (x+y)z'$.

Sol^n:-

$f(x,y,z) = (x+y)z'$

$= xz' + yz'$

$= x(y+y')z + (x+x')yz'$

$= xyz + \underline{xy'z} + xyz' + x'yz'$

using truth table,

x	y	z	(x+y)	z'	f=(x+y).z'
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	1	0	0
0	1	0	1	1	1
0	0	1	0	0	0
0	0	0	0	1	0

for minterm – 1
& maxterm – 0

DN form

$f(x,y,z) = xyz' + xy'z' + x'yz'$.

CN form

$f(x,y,z) = (x+y+z)(x+y'+z)(x'+y+z)(x'+y'+z)(x'+y'+z)$

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Que. Obtain the DNF associated with the Boolean expressions given by $f(x,y,z) = (yz + xz')(xy' + z')'$, by using truth table, and also by using equivalences.

Proof:- The Given Boolean expressions is

$$f(x,y,z) = (yz + xz')(xy' + z')'$$

x	y	z	z'	y'	xy'	$xy'+z'$	$(xy'+z')'$	yz	xz'	$yz+xz'$	$(xy'+z')'\cdot(yz+xz')$
1	1	1	0	0	0	0	1	1	0	1	1
1	1	0	1	0	0	1	0	0	1	1	0
1	0	1	0	1	1	1	0	0	0	0	0
1	0	0	1	1	1	1	0	0	1	1	0
0	1	1	0	0	0	0	1	1	0	1	1
0	1	0	1	0	0	1	0	0	0	0	0
0	0	1	0	1	0	0	1	0	0	0	0
0	0	0	1	1	0	1	0	0	0	0	0

for minterms - 1.

The Required DN form is

$$f(x,y,z) = xyz + x'yz$$

using Algebraic Method or Equivalences

$f(x,y,z) = (yz + xz')(xy' + z')'$

$= (yz + xz')[(xy')' \cdot (z')']$ By De-morgan's law

$= (yz + xz')[(x' + y) \cdot z]$ " & $(a')' = a$

$= (yz + xz')(x'z + yz)$ Distributive law

$= (yz + xz')(yz + x'z)$ commutative law

$= yz + (xz') \cdot (x'z)$ Distributive law

$= yz + (xx')(zz')$ Associative law

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$$= yz + 0.0 \qquad (\because a.a' = 0)$$
$$= 1.yz$$
$$= (x+x')yz$$
$$= xyz + x'yz.$$

Hence

$f(x, y, z) = xyz + x'yz$ is in the DN form.

$(xy'+z')'.(yz+xz')$

1
0
0
0
1
0
0
0

Que. Obtain the DNF associated with the Boolean expressions given by $g(x_1, x_2, x_3) = x_1x_2' + x_3$, by using truth table, and also by using equivalences.

Solⁿ:- using Truth Table →

The Given Boolean Expression

$$g(x_1, x_2, x_3) = x_1x_2' + x_3.$$

x_1	x_2	x_3	x_2'	x_1x_2'	$x_1x_2' + x_3$
1	1	1	0	0	1
1	1	0	0	0	0
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	0	0	1
0	1	0	0	0	0
0	0	1	1	0	1
0	0	0	1	0	0

DNF →

$$g(x_1, x_2, x_3) = x_1x_2x_3 + x_1x_2'x_3 + x_1x_2'x_3' + x_1'x_2x_3 + x_1'x_2'x_3.$$

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using Equivalence laws →

$g(x_1, x_2, x_3) = x_1 x_2' + x_3$

$= x_1 x_2' (x_3 + x_3') + (x_1 + x_1')(x_2 + x_2') x_3$

$= x_1 x_2' x_3 + x_1 x_2' x_3' + (x_1 + x_1')(x_2 x_3 + x_2' x_3)$

$= \underline{x_1 x_2' x_3} + x_1 x_2' x_3' + x_1 x_2 x_3 + x_1' x_2 x_3 +$
$\underline{x_1 x_2' x_3} + x_1' x_2' x_3$.

$(\because a + a = a)$

$= x_1 x_2 x_3 + x_1 x_2' x_3 + x_1' x_2 x_3 + x_1 x_2' x_3' + x_1' x_2' x_3$

conjuctive Normal form (CNF)
OR Canonical Product of Sums (POS)

when a Boolean function f is written as a product of maxterm, it is referred as a conjunctive Normal form or POS

Examples :-

(1) $f(x, y) = (x + y)(x' + y)$

(2.) $f(x, y, z) = (x + y + z)(x + y' + z)(x' + y + z')(x' + y' + z)$.

Complete Conjunctive Normal form! →

- A Boolean function when expressed as a product of all 2^n maxterms of n-variables is called the complete conjunctive Normal form.

Ex! → $f(x, y) = (x + y)(x' + y)(x + y')(x' + y')$. is in complete conjunctive Normal form.

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Ques. 1 Express the Boolean f^n $f(a,b,c) = ab + a'c$ as a product of maxterms.

Sol^n:-

$$f(a,b,c) = ab + a'c$$
$$= (ab + a')(ab + c) \quad [\because x + yz = (x+y)(x+z)]$$
$$= (a + a')(b + a')(a + c)(b + c)$$
$$= (b + a') + (a + c)(b + c) \quad [\because a + a' = 1]$$
$$= (a' + b + c\cdot c') + (a + c + b\cdot b')(b + c + a\cdot a')$$
$$= (a' + b + c)(a' + b + c')(a + c + b)(a + c + b')$$
$$(b + c + a)\cdot(b + c + a').$$

By distributive law

$$= (a + b + c)(a' + b + c)(a + b' + c)(a' + b + c').$$

Ques 2. Obtain the CNF associated with the Boolean expressions given by $f(x,y,z) = (x+y)\cdot z$ by using truth table & Also by using equivalence

Sol^n:- Boolean f^n

$$f(x,y,z) = (x+y)\cdot z.$$

	x	y	z	$x+y$	$f = (x+y)\cdot z$
	1	1	1	1	1
	1	1	0	1	0
	1	0	1	1	1
	1	0	0	1	0
0 – x or y or z	0	1	1	1	1
1 – x' or y' or z'	0	1	0	1	0
	0	0	1	0	0
	0	0	0	0	0

Since the maxterms are Represented by 0 in Boolean f^n $f(x,y,z) = (x+y)\cdot z$ Therefore. The CN form is

$$f(x,y,z) = (x' + y' + z)(x' + y + z)(x + y' + z)(x + y + z')$$
$$(x + y + z)$$

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Ques 3. obtain the CNF associated with the Boolean expressions given by $g(x_1, x_2, x_3) = (x_1 x_3 + x_1 x_3')(x_1 x_2' + x_3)'$, by using Truth table and also by using equivalences.

Sol^n:-

Given Boolean f^n $g(x_1, x_2, x_3) = (x_1 x_3 + x_1 x_3')(x_1 x_2' + x_3)'$

x_1	x_2	x_3	x_2'	x_3'	$x_1 x_3'$	$x_1 x_2'$	$x_1 x_3$	$x_1 x_3 + x_1 x_3'$	$x_1 x_2' + x_3$
1	1	1	0	0	0	0	1	1	1
1	1	0	0	1	1	1	0	1	1
1	0	1	1	0	0	0	1	1	1
1	0	0	1	1	1	1	0	1	1
0	1	1	0	0	0	0	0	0	1
0	1	0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0	0	1
0	0	0	1	1	0	0	0	0	0

$(x_1 x_2' + x_3)'$	$(x_1 x_3 + x_1 x_3')(x_1 x_2' + x_3)'$
0	0
0	0
0	0
0	0
0	0
1	0
0	0
1	0

for maxterm - 0

Hence the CNF of g is as follows

$g(x_1, x_2, x_3) = (x_1 + x_2 + x_3)(x_1 + x_2 + x_3')(x_1 + x_2' + x_3)(x_1 + x_2' + x_3')(x_1' + x_2 + x_3)(x_1' + x_2 + x_3')(x_1' + x_2' + x_3)(x_1' + x_2' + x_3')$

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